9-1 Quadratic Graphs and Their Properties

Content Standards

F.IF.7.a Graph linear and quadratic functions and show intercepts, maxima, and minima.

Also A.CED.2, F.IF.4, F.IF.5, F.BF.1

Objective To graph quadratic functions of the form $y = ax^2$ and $y = ax^2 + c$

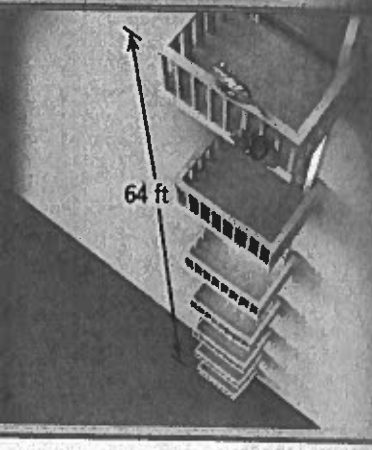
Make sense of the situation. What is the value of h(t) when the flowerpot hits the ground?

MATHEMATICAL PRACTICES

SOLVE IT

Getting Ready!

As a cat walks along the railing of a balcony, it knocks a flowerpot off the railing. The function $h(t) = -16t^2 + c$ gives the height h of the flowerpot after t seconds when it falls from a height of c feet. How long will it take the flowerpot to reach the ground? Explain your reasoning.



Lesson Vocabulary

- quadratic function
- standard form of a quadratic function
- quadratic parent function
- parabola
- axis of symmetry
- vertex
- minimum
- maximum

Recall from Chapter 8 that a polynomial of degree 2, such as $-16x^2 + 64$, is called a quadratic polynomial. You can use a quadratic polynomial to define a *quadratic function* like the one in the Solve It.

Essential Understanding A quadratic function is a type of nonlinear function that models certain situations where the rate of change is not constant. The graph of a quadratic function is a symmetric curve with a highest or lowest point corresponding to a maximum or minimum value.

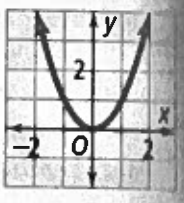
take note

Key Concept Standard Form of a Quadratic Function

A **quadratic function** is a function that can be written in the form $y = ax^2 + bx + c$, where $a \neq 0$. This form is called the **standard form of a quadratic function.**

Examples $y = 3x^2$ $y = x^2 + 9$ $y = x^2 - x - 2$

The simplest quadratic function $f(x) = x^2$ or $y = x^2$ is the **quadratic parent function.**



The graph of a quadratic function is a U-shaped curve called a **parabola.** The parabola with equation $y = x^2$ is shown at the right.

You can fold a parabola so that the two sides match exactly. This property is called *symmetry*. The fold or line that divides the parabola into two matching halves is called the **axis of symmetry.**

The highest or lowest point of a parabola is its **vertex,** which is on the axis of symmetry.

If $a > 0$ in $y = ax^2 + bx + c$, the parabola opens upward.
↓
The vertex is the **minimum** point, or lowest point, of the parabola.

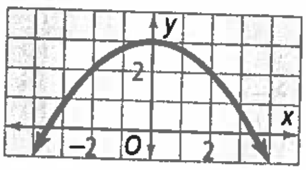
If $a < 0$ in $y = ax^2 + bx + c$, the parabola opens downward.
↓
The vertex is the **maximum** point, or highest point, of the parabola.

Problem 1 Identifying a Vertex

What are the coordinates of the vertex of each graph? Is it a minimum or a maximum?

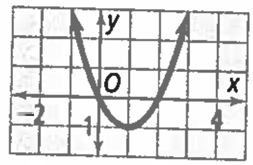
hink
...a parabola have ...th a minimum and ...aximum point?
...parabola either ...upward and has a ...mum point or opens ...ward and has a ...mum point.

A



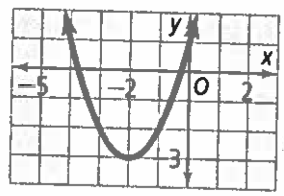
The vertex is (0, 3). It is a maximum.

B



The vertex is $(1, -1)$. It is a minimum.

Got It? 1. What is the vertex of the graph at the right? Is it a minimum or a maximum?



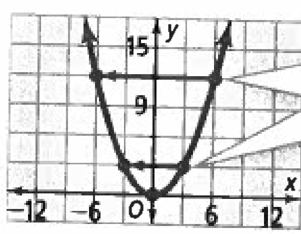
You can use the fact that a parabola is symmetric to graph it quickly. First, find the coordinates of the vertex and several points on one side of the vertex. Then reflect the points across the axis of symmetry. For graphs of functions of the form $y = ax^2$, the vertex is at the origin. The axis of symmetry is the y-axis, or $x = 0$.

Problem 2 Graphing $y = ax^2$

Graph the function $y = \frac{1}{3}x^2$. Make a table of values. What are the domain and range?

...ood values ...for x when ...the table? ...of x that ...by 3 so ...will be

x	$y = \frac{1}{3}x^2$	(x, y)
0	$\frac{1}{3}(0)^2 = 0$	(0, 0)
3	$\frac{1}{3}(3)^2 = 3$	(3, 3)
6	$\frac{1}{3}(6)^2 = 12$	(6, 12)



Reflect the points from the table over the axis of symmetry, $x = 0$, to find more points on the graph.

The domain is all real numbers. The range is $y \geq 0$.

Got It? 2. Graph the function $y = -3x^2$. What are the domain and range?

The coefficient of the x^2-term in a quadratic function affects the width of a parabola as well as the direction in which it opens. When $|m| < |n|$, the graph of $y = mx^2$ is wider than the graph of $y = nx^2$.

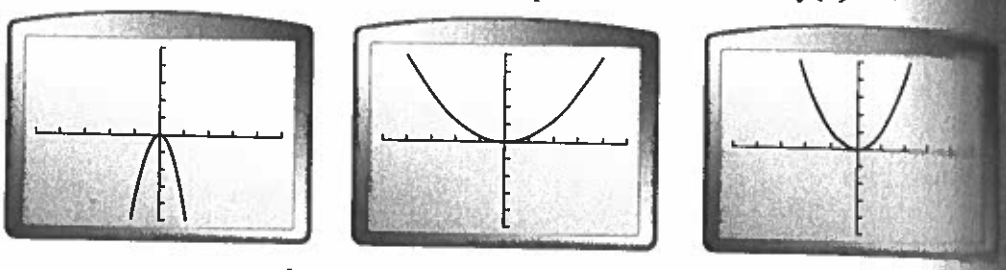
Problem 3 Comparing Widths of Parabolas

Think

Does the sign of the x^2-term affect the parabola's width?
No. The sign of the x^2-term affects only whether the parabola opens upward or downward.

Use the graphs below. What is the order, from widest to narrowest, of the graphs of the quadratic functions $f(x) = -4x^2$, $f(x) = \frac{1}{4}x^2$, and $f(x) = x^2$?

$f(x) = -4x^2$ $f(x) = \frac{1}{4}x^2$ $f(x) = x^2$



Of the three graphs, $f(x) = \frac{1}{4}x^2$ is the widest and $f(x) = -4x^2$ is the narrowest. So, the order from widest to narrowest is $f(x) = \frac{1}{4}x^2$, $f(x) = x^2$, and $f(x) = -4x^2$.

Got It? 3. What is the order, from widest to narrowest, of the graphs of the functions $f(x) = -x^2$, $f(x) = 3x^2$, and $f(x) = -\frac{1}{3}x^2$?

The y-axis is the axis of symmetry for graphs of functions of the form $y = ax^2 + c$. The value of c translates the graph up or down.

Problem 4 Graphing $y = ax^2 + c$

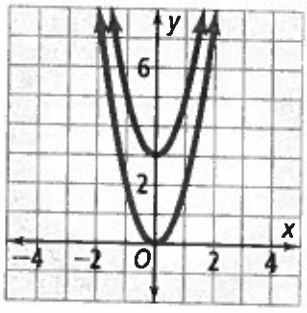
Multiple Choice **How is the graph of $y = 2x^2 + 3$ different from the graph of $y = 2x^2$?**

(A) It is shifted 3 units up.
(B) It is shifted 3 units down.
(C) It is shifted 3 units to the right.
(D) It is shifted 3 units to the left.

Plan

What values should you choose for x?
Use the same values of x for graphing both functions so that you can see the relationship between corresponding y-coordinates.

x	$y = 2x^2$	$y = 2x^2 + 3$
−2	8	11
−1	2	5
0	0	3
1	2	5
2	8	11



The graph of $y = 2x^2 + 3$ has the same shape as the graph of $y = 2x^2$ but is shifted up 3 units. The correct answer is A.

Got It? 4. Graph $y = x^2$ and $y = x^2 - 3$. How are the graphs related?

As an object falls, its speed continues to increase, so its height above the ground decreases at a faster and faster rate. Ignoring air resistance, you can model the object's height with the function $h = -16t^2 + c$. The height h is in feet, the time t is in seconds, and the object's initial height c is in feet.

Problem 5 Using the Falling Object Model

Nature **An acorn drops from a tree branch 20 ft above the ground. The function $h = -16t^2 + 20$ gives the height h of the acorn (in feet) after t seconds. What is the graph of this quadratic function? At about what time does the acorn hit the ground?**

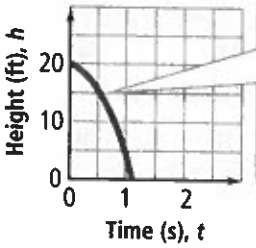
Know
- The function for the acorn's height
- The initial height is 20 ft.

Need
The function's graph and the time the acorn hits the ground

Plan
Use a table of values to graph the function. Use the graph to estimate when the acorn hits the ground.

Think
Can you choose negative values for t? t represents time, so it cannot be negative.

t	$h = -16t^2 + 20$
0	20
0.5	16
1	4
1.5	−16



Graph the function using the first three ordered pairs from the table. Do not plot (1.5, −16) because height cannot be negative.

The acorn hits the ground when its height above the ground is 0 ft. From the graph, you can see that the acorn hits the ground after slightly more than 1 s.

Got It? **5. a.** In Problem 5 above, suppose the acorn drops from a tree branch 70 ft above the ground. The function $h = -16t^2 + 70$ gives the height h of the acorn (in feet) after t seconds. What is the graph of this function? At about what time does the acorn hit the ground?

b. Reasoning What are a reasonable domain and range for the original function in Problem 5? Explain your reasoning.

Lesson Check

Do you know HOW?

parabola. Identify the vertex.

$3x^2$

x^2

$x^2 + 2$

$x^2 - 1$

Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. Vocabulary When is the vertex of a parabola the minimum point? When is it the maximum point?

6. Compare and Contrast How are the graphs of $y = -\frac{1}{2}x^2$ and $y = -\frac{1}{2}x^2 + 1$ similar? How are they different?

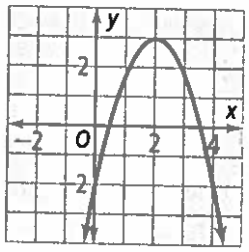
Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

A Practice

Identify the vertex of each parabola. Tell whether it is a minimum or a maximum.

7.



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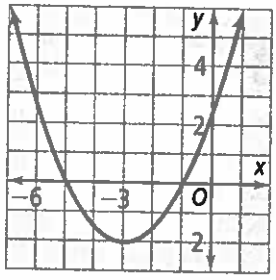


9.

x	y
0	8
1	2
2	0
3	2
4	8

Graph each function. Then identify the domain and range of the function.

10. $y = -4x^2$

11. $f(x) = 1.5x^2$

12. $f(x) = 3x^2$

13. $f(x) = \frac{2}{3}x^2$

14. $y = -\frac{1}{2}x^2$

15. $y = -\frac{1}{3}x^2$

Order each group of quadratic functions from widest to narrowest graph.

16. $y = 3x^2, y = 2x^2, y = 4x^2$

17. $f(x) = 5x^2, f(x) = -3x^2, f(x) = x^2$

18. $y = -\frac{1}{2}x^2, y = 5x^2, y = -\frac{1}{4}x^2$

19. $f(x) = -2x^2, f(x) = -\frac{2}{3}x^2, f(x) = -4x^2$

Graph each function.

20. $f(x) = x^2 + 4$

21. $y = x^2 - 7$

22. $y = \frac{1}{2}x^2 + 2$

23. $f(x) = -x^2 - 3$

24. $y = -2x^2 + 4$

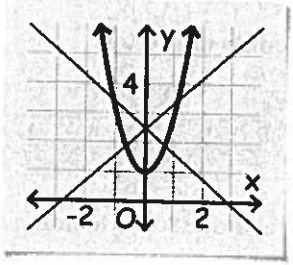
25. $f(x) = 4x^2 - 5$

26. Dropped Object A person walking across a bridge accidentally drops an orange into the river below from a height of 40 ft. The function $h = -16t^2 + 40$ gives the orange's approximate height h above the water, in feet, after t seconds. Graph the function. In how many seconds will the orange hit the water?

27. Nature A bird drops a stick to the ground from a height of 80 ft. The function $h = -16t^2 + 80$ gives the stick's approximate height h above the ground, in feet, after t seconds. Graph the function. At about what time does the stick hit the ground?

B Apply

28. Error Analysis Describe and correct the error made in graphing the function $y = -2x^2 + 1$.



Identify the domain and range of each function.

29. $f(x) = 3x^2 + 6$

30. $y = -2x^2 - 1$

31. $y = -\frac{3}{4}x^2 - 9$

32. $y = \frac{2}{3}x^2 + 12$

33. Writing What information do the numbers a and c give you about the graph of $y = ax^2 + c$?

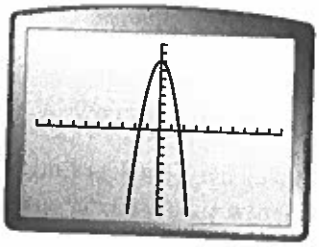
Match each function with its graph.

34. $f(x) = x^2 - 1$

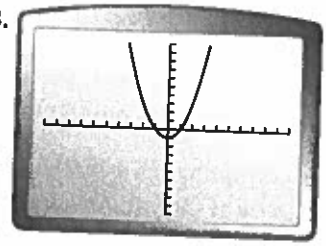
35. $f(x) = -3x^2 + 8$

36. $f(x) = -0.2x^2 + 5$

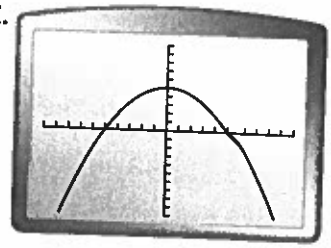
A.



B.



C.



37. Using a graphing calculator, graph $f(x) = x^2 + 2$.

a. If $f(x) = x^2 + 2$ and $g(x) = 3f(x)$, write the equation for $g(x)$. Graph $g(x)$ and compare it to the graph of $f(x)$.

b. If $f(x) = x^2 + 2$ and $h(x) = f(3x)$, write the equation for $h(x)$. Graph $h(x)$ and compare it to the graph of $f(x)$.

c. Compare how multiplying a quadratic function by a number and multiplying the x value of a quadratic function by a number change the graphs of the quadratic functions.

38. Think About a Plan Suppose a person is riding in a hot-air balloon, 154 ft above the ground. He drops an apple. The height h, in feet, of the apple above the ground is given by the formula $h = -16t^2 + 154$, where t is the time in seconds. To the nearest tenth of a second, at what time does the apple hit the ground?

- How can you use a table to approximate the answer between two consecutive whole numbers of seconds?
- How can you use a second table to make your approximation more accurate?

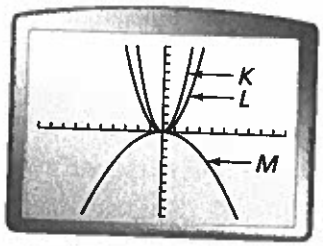
Graphing Calculator **Use a graphing calculator to graph each function. Identify the vertex and axis of symmetry.**

39. $y = \frac{1}{4}x^2 + 3$

40. $f(x) = -1.5x^2 + 5$

41. $y = -3x^2 - 6$

Three graphs are shown at the right. Identify the graph or graphs that fit each description.



42. $a > 0$

43. $a < 0$

44. $|a|$ has the greatest value.

45. $|a|$ has the least value.

STEM **46. Physics** In a physics class demonstration, a ball is dropped from the roof of a building, 72 ft above the ground. The height h, in feet, of the ball above the ground is given by the function $h = -16t^2 + 72$, where t is the time in seconds.

a. Graph the function.

b. How far has the ball fallen from time $t = 0$ to $t = 1$?

c. Reasoning Does the ball fall the same distance from time $t = 1$ to $t = 2$ as it does from $t = 0$ to $t = 1$? Explain.